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Doublet-Triplet Splitting and Fat Branes

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Abstract

We consider the doublet-triplet splitting problem in supersymmetric SU(5) grand unified theory in five dimensions where the fifth dimension is non-compact. We point out that an unnatural fine-tuning of parameters in order to obtain the light Higgs doublets is not required due to the exponential suppression of the overlap of the wave functions.

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The grand unified theory (GUT) [1] is one of the most elegant scenarios in particle physics because of its aesthetic point of view and various interesting physical features. In particular, the success of the gauge coupling unification with the minimal supersymmetric standard model (MSSM) [2] motivates us to consider the supersymmetric (SUSY) GUT seriously. In GUT, there is a serious problem called "the doublet-triplet splitting problem". After GUT symmetry breaking, the Higgs triplets and the Higgs doublets in general obtain the GUT scale mass because these belong to the same multiplet. This requires an unnatural fine-tuning of parameters to obtain the light Higgs doublets [3]. In the minimal SU(5) GUT case, the mass term $MH\bar{H}$ is tuned for the coupling $\bar{H}\Sigma H$ (Σ: the adjoint under SU(5)) to obtain the Higgs doublet with weak scale mass, while leaving the Higgs triplets with GUT scale mass to avoid a fast proton decay by dimension five operators [4]. Many people have tried to solve this problem from various points of view [5]-[15].

Arkani-Hamed and Schmaltz [16] have proposed an interesting mechanism to generate exponentially small coupling in the context of extra dimensions. They discussed that the hierarchies of Yukawa couplings can be explained by the slight displacement of the standard model field wave functions inside four dimensional domain wall in higher dimensional space-time. Even if there is a parameter of order one in the fundamental theory, it is highly suppressed in the effective theory due to the small overlap of wave functions.

In this letter, we apply this mechanism to the doublet-triplet splitting problem. It is pointed out that an unnatural fine-tuning of parameters to obtain the light Higgs doublets is not required in this scenario, *i.e.* at most the tuning of $\mathcal{O}(1)$ orders of magnitude. For simplicity, we consider the SUSY GUT in five dimensions where the fifth dimension is non-compact. The action of the Higgs sector is

$$\begin{aligned} S &= \int d^4x dy \Big[\int d^4\theta (H^\dagger e^{-V} H + H^{c\dagger} e^V H^c + \bar{H}^\dagger e^V \bar{H} + \bar{H}^{c\dagger} e^{-V} \bar{H}^c) \\ &+ \Big\{ \int d^2\theta \left(H^c(\partial_y + X(y) + M) H + \bar{H}^c(\partial_y + \bar{X}(y) + \bar{M}) \bar{H} \right) \\ &+ \delta(y) \int d^2\theta \left(\lambda_1 \mathrm{tr}(X^2\Sigma) + \lambda_2 \mathrm{tr}(\bar{X}^2\Sigma) + \lambda_3 \mathrm{tr}(X\Sigma^2) + \lambda_4 \mathrm{tr}(\bar{X}\Sigma^2) + \frac{1}{2} m_0 \mathrm{tr}(\Sigma^2) \right) + \mathrm{h.c.} \Big\} \Big], \end{aligned} \tag{1}$$

where $H(\bar{H}), H^c(\bar{H}^c)$ are left-handed (charge conjugated right-handed) chiral $\mathcal{N}=1$ in four dimensional superfield components of the single $\mathcal{N}=1$ in five dimensional chiral superfield $H(\mathbf{5}) = \left(H, \bar{H}^c\right)$ and $\bar{H}(\bar{\mathbf{5}}) = \left(\bar{H}, H^c\right)$. $\mathbf{5}, \bar{\mathbf{5}}$ are the representations of SU(5). $X(y), \bar{X}(y)$ are the bulk fields in the $\mathbf{24}$ dimensional representation under $SU(5)$. [2] Σ is an usual SU(5) GUT adjoint Higgs field, which is assumed to be localized on the brane at $y=0$. The fifth dimensional coordinate is denoted by y. We assume that $X(y), \bar{X}(y)$ depends on y, and $M, \bar{M}$ do not. $\lambda_{1\sim 4}$ are dimesionless constants and m_0 is a bare mass parameter. This formulation of the action Eq. (1) is useful because it is written by using the $\mathcal{N}=1$ superfield formalism and $\mathcal{N}=1$ SUSY is manifest [17, 18].

F-flatness conditions of $X, \bar{X}$ and Σ are

$$0 = \frac{\partial W}{\partial X} = H^c H - \frac{1}{5} \mathrm{tr}(H^c H)$$

[2] $X(y)$ and $\bar{X}(y)$ are rescaled by $M_*^{-1/2}$, where M_* is the Planck scale in five dimensional theory, since their mass dimension is 3/2 in five dimensions.

$$
\begin{aligned}
& +\delta(y)\left\{2\lambda_1 X\Sigma+\lambda_2\Sigma^2-\frac{1}{5}\left(2\lambda_1\mathrm{tr}(X\Sigma)+\lambda_2\mathrm{tr}(\Sigma^2)\right)\right\}, && (2)\\
0 \;=\; & \frac{\partial W}{\partial \bar{X}}=\bar{H}^c\bar{H}-\frac{1}{5}\mathrm{tr}(\bar{H}^c\bar{H}) \\
& +\delta(y)\left\{2\lambda_3 \bar{X}\Sigma+\lambda_4\Sigma^2-\frac{1}{5}\left(2\lambda_3\mathrm{tr}(\bar{X}\Sigma)+\lambda_4\mathrm{tr}(\Sigma^2)\right)\right\}, && (3)\\
0 \;=\; & \frac{\partial W}{\partial \Sigma}=\delta(y)\left\{\lambda_1 X^2+\lambda_2\bar{X}^2+2\lambda_3 X\Sigma+2\lambda_4\bar{X}\Sigma\right.\\
& \left.+m_0\Sigma-\frac{1}{5}\left(\lambda_1\mathrm{tr}(X^2)+\lambda_2\mathrm{tr}(\bar{X}^2)+2\lambda_3\mathrm{tr}(X\Sigma)+2\lambda_4\mathrm{tr}(\bar{X}\Sigma)\right)\right\}, && (4)
\end{aligned}
$$

where we omitted SU(5) indices for convenience. The trace part is proportional to the unit matrix. The solutions of Eqs. (2) and (3) are

$$
\begin{aligned}
& H^cH-\frac{1}{5}\mathrm{tr}(H^cH)=0, && (5)\\
& 2\lambda_1 X(0)+\lambda_3\Sigma=0, && (6)\\
& \bar{H}^c\bar{H}-\frac{1}{5}\mathrm{tr}(\bar{H}^c\bar{H})=0, && (7)\\
& 2\lambda_3 \bar{X}(0)+\lambda_4\Sigma=0. && (8)
\end{aligned}
$$

It is remarkable that Eqs. (6) and (8) connect the vacuum expectation values (VEVs) $X(0)$ and $\bar{X}(0)$ in the bulk with $\langle\Sigma\rangle$ on the brane at $y=0$. As we will see later, y-independent masses of Higgs (*i.e.* $X(0)+M$ and $\bar{X}(0)+\bar{M}$) determine the coordinates which Higgs wave functions are localized. These masses are different between the Higgs triplet and the Higgs doublet since $\langle X(0)\rangle$ and $\langle\bar{X}(0)\rangle$ are proportional to $\langle\Sigma\rangle$. Therefore, the splitting occurs naturally. Although a similar model has been considered in Ref. [10, 11], they simply assumed that $\langle X(0)\rangle$ and $\langle\bar{X}(0)\rangle$ take the form proportional to $\langle\Sigma\rangle$. On the other hand, we *derived* this from the equations of motion. [3] This is a crucial difference between Ref. [11] and this paper. Using Eqs. (6) and (8), Eq. (4) reproduces the stationary condition of the Higgs potential in the minimal SU(5) GUT,

$$
0=-\frac{3}{4}\left(\frac{\lambda_3^2}{\lambda_1}+\frac{\lambda_4^2}{\lambda_2}\right)\left\{\Sigma^2-\frac{1}{5}\mathrm{tr}(\Sigma^2)\right\}+m_0\Sigma. \tag{9}
$$

Furthermore, substituting $\langle\Sigma\rangle=\mathrm{diag}(2,2,2,-3,-3)\sigma$, where σ is a constant, we obtain

$$
\frac{3}{2}\left(\frac{\lambda_3^2}{\lambda_1}+\frac{\lambda_4^2}{\lambda_2}\right)\sigma+2m_0=0. \tag{10}
$$

Expanding the five dimensional superfields $H, H^c, \bar{H}$ and $\bar{H}^c$ by the mode functions as

$$
\begin{aligned}
H(x,y) &= \sum_n \phi_n(y)H_n(x), && (11)\\
H^c(x,y) &= \sum_n \phi^c_n(y)H^c_n(x), && (12)
\end{aligned}
$$

[3] The author would like to thank T. Yanagida for suggesting that $\langle X(0)\rangle$ and $\langle\bar{X}(0)\rangle$ should be derived from the potential. In Ref. [10], this point is not accomplished.

$$\bar{H}(x,y) = \sum_n \bar{\phi}_n(y)\bar{H}_n(x), \tag{13}$$

$$\bar{H}^c(x,y) = \sum_n \bar{\phi}^c_n(y)\bar{H}^c_n(x), \tag{14}$$

where x denotes the coordinate of the four dimensional space-time. The equations of motions for the zero mode wave functions of Higgs fields are

$$(\partial_y + X(y) + M)\ \phi_0(y) = 0, \tag{15}$$

$$(-\partial_y + X(y) + M)\ \phi^c_0(y) = 0, \tag{16}$$

$$(\partial_y + \bar{X}(y) + \bar{M})\ \bar{\phi}_0(y) = 0, \tag{17}$$

$$(-\partial_y + \bar{X}(y) + \bar{M})\ \bar{\phi}^c_0(y) = 0. \tag{18}$$

Let us assume for simplicity that $X(y) = X(0) + a^2 y$, $\bar{X}(y) = \bar{X}(0) + a^2 y$ in a small region of the point crossing zero. a is a constant of mass dimension one. These mass functions generate Gaussian zero mode wave functions. The zero mode wave functions take the following form,

$$\phi_0(y) \sim \exp\left\{-\frac{a^2}{2}\left(y - \frac{X(0)+M}{a^2}\right)^2\right\}, \tag{19}$$

$$\phi^c_0(y) \sim \exp\left\{\frac{a^2}{2}\left(y - \frac{X(0)+M}{a^2}\right)^2\right\}, \tag{20}$$

$$\bar{\phi}_0(y) \sim \exp\left\{-\frac{a^2}{2}\left(y - \frac{\bar{X}(0)+\bar{M}}{a^2}\right)^2\right\}, \tag{21}$$

$$\bar{\phi}^c_0(y) \sim \exp\left\{\frac{a^2}{2}\left(y - \frac{\bar{X}(0)+\bar{M}}{a^2}\right)^2\right\}. \tag{22}$$

Since the wave functions $\phi^c_0(y)$ and $\bar{\phi}^c_0(y)$ are not normalizable, its normalization constants must be zero. This result is consistent with Eqs. (5) and (7).

Now, we consider two cases which realize the doublet-triplet splitting. One is achieved through the bulk Higgs mass term [10, 11] and the other is achieved through the coupling of the singlet and the Higgs fields [11]. First, we will show that the former case cannot incorporate the hierarchy of Yukawa couplings although the doublet-triplet splitting occur. The Higgs mass term in five dimensions is

$$\begin{aligned} &\int d^4x dy \int d^2\theta M_* H(x,y)\bar{H}(x,y) \\ = \;& M_* \int dy \sqrt{\frac{a^2}{2\pi}} \exp\left\{-\frac{a^2}{2}\left(y - \frac{X(0)+M}{a^2}\right)^2 - \frac{a^2}{2}\left(y - \frac{\bar{X}(0)+\bar{M}}{a^2}\right)^2\right\} \\ &\times \int d^4x \int d^2\theta H_0(x)\bar{H}_0(x). \end{aligned} \tag{23}$$

Higgs mass in four dimensions can be read by integrating out degrees of freedom in the fifth dimension,

$$\frac{M_*}{\sqrt{2}} \exp\left\{-\frac{(X(0)+M-\bar{X}(0)-\bar{M})^2}{4a^2}\right\} \int d^4x \int d^2\theta H_0(x)\bar{H}_0(x). \tag{24}$$

The masses of the Higgs triplets and the Higgs doublets are

$$M_3 \sim M_* \exp\left[-\frac{\{2(x-\bar{x})M_* + (m-\bar{m})M_*\}^2}{4a^2}\right] \gtrsim M_{\rm GUT} \simeq 10^{16}\ {\rm GeV}, \qquad (25)$$

$$M_2 \sim M_* \exp\left[-\frac{\{-3(x-\bar{x})M_* + (m-\bar{m})M_*\}^2}{4a^2}\right] \simeq M_W \simeq 10^{2}\ {\rm GeV}, \qquad (26)$$

where $x, \bar{x}$ and $m, \bar{m}$ are defined as follows,

$$\begin{aligned} X(0) &= x\ {\rm diag}(2,2,2,-3,-3)M_*, &(27)\\ \bar{X}(0) &= \bar{x}\ {\rm diag}(2,2,2,-3,-3)M_*, &(28)\\ M &= m\ M_*, \quad \bar{M} = \bar{m}\ M_*. &(29)\end{aligned}$$

We assumed here that the order of the VEV's of $X(0), \bar{X}(0)$ and $M, \bar{M}$ are around the five dimensional Planck scale M_*.

Before discussing the doublet-triplet splitting in detail, various scales in our model are summarized. There are three typical mass scales, *i.e.* the five dimensional Planck scale M_*, the wall thickness scale L^{-1} which should be considered as the compactification scale and the inverse width of Gaussian zero modes a^{-1}. As explained in Ref. [16], for the description to make sense, the wall thickness L should be larger than the inverse width of Gaussian zero modes a^{-1}. Furthermore, a^{-1} should be larger than or equal to the five dimensional Planck length M_*^{-1},

$$L^{-1} < a \leq M_*. \qquad (30)$$

We take L^{-1} to be M_{GUT} in order to preserve the gauge coupling unification. The five dimensional Planck scale M_* can be taken to be about 10^{17}GeV or 10^{18} GeV from the above relation. Hereafter, $M_* \simeq 10^{18}$ GeV is taken for simplicity. In this case, the masses of the Higgs triplets (25) and the Higgs doublets (26) become

$$\exp\left[-\frac{\{2(x-\bar{x})+m-\bar{m}\}^2}{4}\right] \gtrsim 10^{-2}, \qquad (31)$$

$$\exp\left[-\frac{\{-3(x-\bar{x})+m-\bar{m}\}^2}{4}\right] \simeq 10^{-16}, \qquad (32)$$

where $a \simeq M_*$ is assumed for simplicity. These can be easily solved as

$$\begin{aligned} -3.2844\cdots \lesssim &\quad x-\bar{x} &\lesssim -1.5685\cdots, \qquad (33)\\ 2.2793\cdots \lesssim &\quad m-\bar{m} &\lesssim 7.4276\cdots. \qquad (34)\end{aligned}$$

This means that the doublet-triplet splitting is realized by $\mathcal{O}(1)$ tuning of parameters in contrast to an unnatural $\mathcal{O}(10^{14})$ fine-tuning of parameters in four dimensional case. As mentioned above, however, this case cannot reproduce the correct orders of magnitude of Yukawa couplings. [4] In order to show this, we discuss $x - \bar{x} \simeq -3$ and $m - \bar{m} \simeq 3$

[4] There are several attempts to explain fermion mass hierarchy in the fat brane approach [19, 20]. The differences between Refs. [19, 20] and this paper are the following. In Ref. [19], the wave functions of Higgs fields are flat in extra dimensions and non-supersymmetric case is considered. In Ref. [20], the wave functions of the matter are localized at the different points generation by generation.

case as an example. In this case, the Higgs triplets $H_3, \bar{H}_3$ are localized at $y \simeq (2x + m)M_*^{-1}, (2x+m+3)M_*^{-1}$ and the Higgs doublets $H_2, \bar{H}_2$ are localized at $y \simeq (-3x + m)M_*^{-1}, (-3x+m-12)M_*^{-1}$, respectively. Note that the relative distance between H_2 and $\bar{H}_2$ is large. This is the problem. The left-handed quark superfield couples to both H_2 and $\bar{H}_2$. In order to obtain $\mathcal{O}(1)$ top Yukawa coupling, the left-handed quark superfield of the third generation Q_3 and the right-handed quark superfield of the third generation U_3^c must be localized around H_2. We will show that the correct order of magnitude of the bottom Yukawa coupling cannot be reproduced in this situation. The top Yukawa couplings in five dimensions are written by

$$\int d^4xdy \int d^2\theta \frac{Y_t}{\sqrt{M_*}}\, Q_3(x,y)U_3^c(x,y)H_2(x,y) + \text{h.c.} \tag{35}$$

$$\simeq \frac{Y_t}{\sqrt{M_*}}\left(\frac{2M_*^2}{\pi}\right)^{3/4} \int dy e^{-M_*^2(y-y_{q_3})^2} e^{-M_*^2(y-y_{u_3^c})^2} e^{-M_*^2(y-y_{h_2})^2}$$
$$\times \int d^4xd^2\theta\; Q_{3,0}(x)U_{3,0}^c(x)H_{2,0}(x) + \text{h.c}, \tag{36}$$

where Y_t is a top Yukawa coupling constant of order unity in five dimensions. We assumed that the zero mode wave functions of Q_3, U_3^c and H_2 are also Gaussian and localized at $y \sim y_{q_3}, y_{u_3^c}$ and y_{h_2}, respectively. The effective top Yukawa coupling in four dimensions y_t can be read as

$$y_t \sim Y_t \exp\left[-\frac{1}{3}M_*^2\left\{(y_{q_3}-y_{u_3^c})^2 + (y_{q_3}-y_{h_2})^2 + (y_{u_3^c}-y_{h_2})^2\right\}\right]. \tag{37}$$

To be $y_t \sim \mathcal{O}(1)$, $y_{q_3} \simeq y_{u_3^c} \simeq y_{h_2}$.

On the other hand, the effective bottom Yukawa coupling in four dimensions are obtained by replacing Y_t with Y_b and $y_{u_3^c}, y_{h_2}$ with $y_{d_3^c}, y_{\bar{h}_2}$,

$$y_b \sim Y_b \exp\left[-\frac{1}{3}M_*^2\left\{(y_{q_3}-y_{d_3^c})^2 + (y_{q_3}-y_{\bar{h}_2})^2 + (y_{d_3^c}-y_{h_2})^2\right\}\right], \tag{38}$$

$$\sim Y_b \exp\left[-\frac{1}{3}M_*^2\left\{(y_{h_2}-y_{d_3^c})^2 + (y_{h_2}-y_{\bar{h}_2})^2 + (y_{d_3^c}-y_{\bar{h}_2})^2\right\}\right], \tag{39}$$

$$\lesssim Y_b \exp\left[-\frac{1}{3}M_*^2(y_{h_2}-y_{\bar{h}_2})^2\right] \simeq \exp(-48) \simeq 10^{-21}, \tag{40}$$

where Y_b is a bottom Yukawa coupling constant of order unity, and $y_{q_3} \simeq y_{h_2}$ is used in the second line. Clearly, this is not realistic. Even if we take the other values satisfying Eqs. (33) and (34) as $x - \bar{x}$ and $m - \bar{m}$, this result is not changed.

In order to improve this point, the Higgs triplets and the Higgs doublets are not only localized separately, but also the same multiplets have to be closely localized each other. Furthermore, the doublet-triplet splitting has to be realized by the overlap between the Higgs fields and the other bulk field, and by localizing the Higgs triplets close to this bulk field. This can be simply achieved by introducing the singlet field in the bulk [11]. [5]

[5] A similar coupling is considered, but the mechanisms to localize the bulk singlet are different between Ref. [11] and this paper.

The action of the singlet sector is based on the "shining" mechanism [17]

$$S = \int d^4xdy \left[\int d^4\theta(S^\dagger S + S^{c\dagger}S) + \left\{\int d^2\theta S^c(\partial_y + m_s)S - \delta(-y)\int d^2\theta JS + \text{h.c.}\right\}\right], \tag{41}$$

where S is an SU(5) singlet superfield in the bulk, S^c is its conjugated superfield, J is a constant source and m_s is a mass parameter. F-flatness conditions are

$$0 = (\partial_y + m_s)S, \tag{42}$$

$$0 = (-\partial_y + m_s)S^c - J\delta(-y). \tag{43}$$

The normalizable solutions of Eqs. (42) and (43) are

$$S = 0, \tag{44}$$

$$S^c = \theta(-y)Je^{m_s y}, \tag{45}$$

where $\theta(y)$ is a step function for y.
The doublet-triplet splitting can be accomplished by introducing the following coupling. [6]

$$\frac{1}{\sqrt{M_*}}\int d^4xdy\left\{\int d^2\theta S^c(x,y)H(x,y)\bar{H}(x,y) + \text{h.c.}\right\} \tag{46}$$

$$= \frac{1}{\sqrt{M_*}}\int dyS^c(y)\phi_0(y)\bar{\phi}_0(y)\int d^4xd^2\theta H_0(x)\bar{H}_0(x) + \text{h.c.} \tag{47}$$

$$= \frac{M_*}{2\sqrt{2}}\exp\left[-\frac{1}{2M_*^2}\left\{(X(0)+M)^2 + (\bar{X}(0)+\bar{M})^2\right\} + \frac{(X(0)+M+\bar{X}(0)+\bar{M}+m_s)^2}{4M_*^2}\right]$$

$$\times\int d^4xd^2\theta H_0(x)\bar{H}_0(x) + \text{h.c.}, \tag{48}$$

where we assumed $J \simeq M_*^{3/2}, a \simeq M_*$. [7] Therefore, the masses of the Higgs triplets and the Higgs doublets are

$$M_3 \simeq \frac{M_*}{2\sqrt{2}}\exp\left[-\frac{1}{2}\{(2x+m)^2 + (2\bar{x}+\bar{m})^2\} + \frac{1}{4}(s+2x+m+2\bar{x}+\bar{m})^2\right], \tag{49}$$

$$M_2 \simeq \frac{M_*}{2\sqrt{2}}\exp\left[-\frac{1}{2}\{(-3x+m)^2 + (-3\bar{x}+\bar{m})^2\} + \frac{1}{4}(s-3x+m-3\bar{x}+\bar{m})^2\right], \tag{50}$$

where we defined $m_s \equiv sM_*$.

If we consider the case that the Higgs triplets are localized at $y \simeq 0$, *i.e.* $2x+m \simeq 0$ and $2\bar{x}+\bar{m} \simeq 0$ for simplicity, the conditions of the doublet-triplet splitting are

$$M_3 \sim \frac{M_*}{2\sqrt{2}}\exp(s^2/4) \gtrsim M_{\text{GUT}} \sim 10^{16}\ \text{GeV}, \tag{51}$$

$$\begin{aligned} M_2 &\sim \frac{M_*}{2\sqrt{2}}\exp\left[-\frac{1}{2}\{(-5x)^2 + (-5\bar{x})^2\} + \frac{1}{4}(s-5x-5\bar{x})^2\right] \\ &\simeq M_W \simeq 10^2\ \text{GeV}. \end{aligned} \tag{52}$$

[6] A similar coupling was considered in Ref. [11]. They simply assumed the VEV of the singlet to be the GUT scale and does not specify the mechanism to generate the VEV.

[7] In order for the bulk Higgs mass term not to be allowed, we have to impose a symmetry, for example, an R-symmetry.

If we consider the case with $x = \bar{x}$, one of the solutions of Eq. (52) is $x = \bar{x} \simeq 7$ and $s \simeq \sqrt{2}$. In this case, the mass of Higgs triplets is $M_3 \simeq 0.6 M_*$. Eq. (51) is satisfied since we are taking M_* to be 10^{18} GeV. The Higgs triplets are localized at $y \simeq 0$, and the Higgs doublets are localized at $y \simeq -35 M_*^{-1}$. The doublet-triplet splitting is realized by $\mathcal{O}(1)$ tuning of the parameters in contrast to an unnatural fine-tuning in four dimensional case.

The next question is whether the following Yukawa coupling hierarchy can be obtained from the above setup [8];

$$\begin{array}{lll} y_t \sim \mathcal{O}(1), & y_c \sim \mathcal{O}(10^{-2}), & y_u \sim \mathcal{O}(10^{-5}), \\ y_b \sim \mathcal{O}(10^{-2}), & y_s \sim \mathcal{O}(10^{-4}), & y_d \sim \mathcal{O}(10^{-5}), \\ y_\tau \sim \mathcal{O}(10^{-2}), & y_\mu \sim \mathcal{O}(10^{-4}), & y_e \sim \mathcal{O}(10^{-6}). \end{array}$$

We would like to find from Eq. (37) the coordinates where the zero mode wave functions of the matter fields are localized and which induces the above hierarchy. We also take into account that the coefficients of the dimension five operators induced by the Planck scale physics $\frac{1}{M_P} QQQL$, that is $\frac{M_P}{M_*}\, e^{-(M_* r)^2}$ where r is the distance between the wave functions of quarks and the leptons, have to be less than 10^{-7} to keep the nucleon stable enough as required by experiments [22]. This constraints can be satisfied if $r \gtrsim (4 \sim 5)\ M_*^{-1}$. The typical solution we found is

$$y_{h_2} \simeq y_{\bar{h}_2} \simeq y_{q_3} \simeq y_{u_3^c} \sim -35 M_*^{-1},\ y_{q_2} \simeq y_{u_2^c} \simeq y_{d_3^c} \simeq -37.6 M_*^{-1}, \tag{53}$$

$$y_{q_1} \simeq y_{u_1^c} \simeq y_{d_1^c} \simeq -39.1 M_*^{-1},\ y_{d_2^c} \simeq -38.7 M_*^{-1},\ y_{l_3} \simeq -33 M_*^{-1}, \tag{54}$$

$$y_{e_3^c} \simeq -32.4 M_*^{-1},\ y_{l_2} \simeq y_{e_2^c} \simeq -31.3 M_*^{-1},\ y_{l_1} \simeq y_{e_1^c} \simeq -30.4 M_*^{-1}. \tag{55}$$

We have checked that this configuration also satisfies the constraints for the coefficients of the dimension five operator $U^c U^c U^c E^c$.

In summary, we have discussed the doublet-triplet splitting problem in SUSY SU(5) GUT in five dimensions where the fifth dimension is non-compact. It was pointed out that an unnatural fine-tuning of parameters in order to obtain the light Higgs doublets is not required due to the exponential suppression of the overlap of the wave functions. We have found the explicit configuration of the Higgs and matter wave functions that realizes the doublet-triplet splitting, satisfies the constraints for the proton decay due to the dimension five operators induced by the Planck scale physics as well as by the Higgs triplet exchange and generates the correct orders of magnitude of Yukawa couplings. Furthermore, the gauge coupling unification is preserved because the inverse width of the fat brane L^{-1} is the GUT scale.

Although the order of Yukawa couplings are explained, it is important to investigate whether the mixing angles can also be explained. Also, it is easy to incorpolate SUSY breaking in our setup (see Refs. [17, 20, 21] due to the shining mechanism and Ref. [23] due to the coexistence of BPS domain walls.). It is very interesting to study the spectrum of the soft SUSY breaking terms in our setup, and investigate whether these spectrum satisfy the various experimental bounds. We leave these issues for future work.

[8] We simply neglect the neutrino sector and the mixing angles since the detailed analysis of the fermion mass hiearchy and their mixing angles in our setup is not a main subject in this paper.

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